Regularization Preserving Localization of Close Edges

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***Abstract*—In this letter, we address the problem of the influence of neighbor edges and their effect on the edge delocalization while extracting a neighbor contour by a derivative approach. The properties to be fulfilled by the regularization operators to minimize or suppress this side effect are deduced, and the best detectors are pointed out. The study is carried out in 1-D for discrete signal. We show that among the derivative filters, one of them can correctly detect our model edges without being influenced by a neighboring transition, whatever their separation distance is and their respective amplitude is. A model of contour and close transitions is presented and used throughout this letter. The noise effect on the edge delocalization is recalled through one of the Canny criteria. Different derivative filters are applied onto synthetic images, and their performances are compared.**

***Index Terms*—Edge detection, edge localization, edge model, neighbor edge, regularization filter.**

I. Introduction

SIGNAL or image regularization is a crucial stage in the interpretation process and especially in all "high-pass" processing like edge detection [1]. Some work dealing with this topic has been conducted proposing solutions to preserve edge shape [2] and localization while regularizing in noisy images [3]–[6]. Our work is focused on derivative contour approach. This is a quite challenging task, and in most classical edge detection algorithms, some delocalization occurs when edges are close one to the other, which is very common in real images. It is this particular point that is studied in this letter. How does the regularization matter on edge localization when taking into account the neighbor edges? Can two close contours be separately detected without any delocalization due to the regularization filter? An iterative solution was proposed by Shen *et al.* [7]. Their solution first detects coarse contours; then, by including the neighborhood, the zero transitions associated with their amplitude, an accurate solution is reached. We herein present a single-pass filtering that does not affect the contour localization, no matter the influence of the neighbor contour. Dimensional control, image correspondence, for middle range signal-to-noise ratio (SNR) images (weak additive noise) are among the applications that could benefit from this filtering.

Micheli *et al.* [8] showed that, on weakly noisy images, because of the filter sampling, the shape of narrow-width filters did not affect the result. We are here presenting a discrete approach that shows the importance of the filter shape while taking into account the mutual influence of the two contours. Finally, Canny's localization criterion [9]–[12] with noise is valid. Demigny criterion in discrete domain is recalled [13].

The first part presents the classical contour model onto which a neighbor contour is added. This situation leads to two configurations, (additive influence and subtractive influence), with each of them creating mutual influence between the two contours. For each configuration, the properties required by the filter to avoid delocalization due to mutual influence of the contours are expressed. These properties are studied for the major well-known derivative operators. Finally, the 2-D extension is presented through a simple example for two regularization filters, among which only one possesses the right properties regarding the non-delocalization by adjacent contour.

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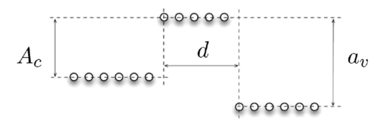

Fig. 1. Discrete model with two steps ($a_v < 0$).

II. Definitions and Notations

A. Two Steps Edge Model

A simple model for two neighbor edges is proposed; two Heaviside steps separated by a distance d corrupted by a Gaussian white additive noise

$$y_k = A_c Y_k + a_v Y_{k-d} + \sigma \eta_k \tag{1}$$

where Y is Heaviside function, if $k \geq 0$ $Y_k = 1$, else $Y_k = 0$; $A_c \geq 0$ is the amplitude of the step at the current point; a_v is the variable amplitude of the neighbor step localized at a distance d from the current point; and η is a stochastic variable denoting normalized amplitude of a Gaussian white noise of 0 mean and standard deviation of 1 (σ is noise standard deviation). Fig. 1 illustrates the two steps edge discrete model without noise.

B. Edge Localization by Derivative Approach

The localization of a luminance transition modeled by a discrete signal y can be obtained by detecting the minimum (or the maximum) of the discrete derivative u of y. As it is well known, differentiating a discrete signal is an ill-posed problem and an estimate must be performed from a regularized version of y. If the impulse response of the regularization filter is denoted by h, the regularized version of y, denoted by r, can be written as $r = h * y$. The derivative of r is given by

$$u = (h * y)' = f * y \tag{2}$$

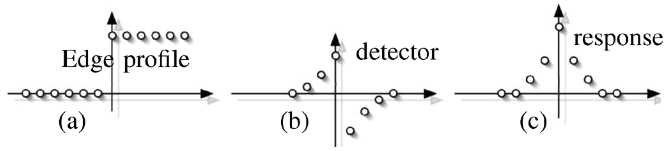

Fig. 2. (a) Discrete signal with one step. (b) Regularized derivative estimator. (c) Estimate of the derivative (denoted also as h).

where f is the derivative of h. The half-width of the impulse response will be further denoted by L. The numerical derivative is defined by

$$f'_k = f_k - f_{k-1}. \tag{3}$$

This definition is consistent with (2). Fig. 2 shows a discrete signal with one step, the regularized derivative estimator f, and the estimate of the derivative u. It is to be noted that we choose to define all the derivative filters without 0 at center point so that only one maximum (or minimum) is detected for a stepwise edge. Therefore, the regularizing filter h is even (null phase).

C. Edge Localization Equation

The regularized derivative of the signal defined in (1) can be estimated by $u_k = A_c h_k + a_v h_{k-d} + \sigma \sum_{n=-\infty}^{+\infty} \eta_{k-n} h'_n$. Three parts are pointed out that model in a simple but complete way the detection features of image luminance edges. The first term is the expected output; the second one is the neighbor edge bearing; and the third one is the unavoidable noise contribution. The main purposes of this letter are studying the consequences of the last two terms on the first one and deducing the properties of the regularization filter h minimizing those bearing. We conclude this part in the next section with a definition about the detection and localization of an edge.

D. Edge Presence and Localization

A transition (edge) of amplitude $A_c > 0$ does exist and is localized at point of abscissa k if the derivative signal has the following properties:

$$\begin{cases} u_{k-1} < u_k \quad \text{and} \quad u_k > u_{k+1} \\ u_k > 0. \end{cases} \tag{4}$$

These relations are quite natural, and they correspond to the definition of a local maximum for a positive transition. However, as the signal is embedded in noise, we will have to introduce a threshold T in the last (4).

III. Properties of the Regularizing Operator

The regularization aims at giving the best possible representation of the signal whatever the corrupting noise. In the case of luminance transitions (edges), preserving the localization is the most important point.

A. Under Noise Influence: $a_v = 0$, $\sigma \neq 0$

Noise can influence transition localization in the regularized signal r. Extrema of the derivative u give the positions of these transitions. Actually, as noise corrupts signal and its derivative, it may delocalize the detection point. The results in this case are well known, and we recall the equation that can be considered as a discrete version of the Canny's localization criterion [14] for a noise of unitary variance: $L(f) = (|f_0| / \sum_{l=-L+1}^{L} f_l'^2)$. This criterion means that variation around maximum of detection has to be the largest possible, while the relative noise variation must stay at the lowest possible level. Doing that allows to limit the influence of the noise on localization.

B. Under the Influence of a Close-By Transition: $a_v \neq 0$, $\sigma = 0$

The response to a transition of amplitude A_c standing next to a close-by transition a_v is now to be considered. Noise influence does not have to be taken into account, for the previous section has dealt with this kind of perturbation. Transition of amplitude A_c being localized at $k = 0$ in the image, it is the influence of the neighbor transition a_v on the localization of the detected edge that is under study.

1) Subtractive Influence ($a_v < 0$): Subtractive influence is due to a neighbor edge with amplitude of opposite sign: $u_k = A_c h_k - |a_v| h_{k-d}$. As this influence is subtractive, it results in a decrease of the response amplitude, and it tends to delocalize this response. Derivative at $k = 0$ gives the delocalization sign: $u'_0 = -|a_v| h'_{-d}$. Derivative being negative, the maximum corresponding to the detected edge occurs at $k \leq 0$. Therefore, generally speaking, distance between consecutive opposite detected edges tends to be enlarged due to this subtractive influence. Criteria given by (4) for an edge of amplitude A_c to be existing at $k = 0$ can be simplified as follows:

$$\begin{cases} u_{-1} < u_0 \\ u_0 > 0 \end{cases} \tag{5}$$

leading to $\begin{cases} v_d(h_0 - h_{-1}) > h_{-d} - h_{-d-1} \\ v_d h_0 > h_{-d} \end{cases}$, where $v_d = (A_c / |a_v|)$. While the first relation in (5) gives the condition for edge localization at $k = 0$, the second has to be verified for the transition to exist at $k = 0$. Equation (5) can be explained under the following form [see also Fig. 2 and the discrete definition of the derivative in (3)]:

$$\begin{cases} v_d h'_0 > h'_{-d}, & \text{non-delocalization condition} \\ v_d h_0 > h_{-d}, & \text{existence condition.} \end{cases} \tag{6}$$

Writing $V_{\text{non-deloc.}} = h'_{-d}/h'_0$ and $V_{\text{existence}} = h_{-d}/h_0$, it comes

$$\begin{cases} V_{\text{non-deloc.}} < v_d \\ V_{\text{existence}} < v_d. \end{cases} \tag{7}$$

The non-delocalization condition must be fulfilled as long as the transition is detected (existence condition verified, otherwise delocalization always occurs). It follows that if one of the abovementioned conditions is not verified, it is the existing one that is to be overcome to avoid any delocalization problem. Following this strategy leads to a situation where all the detected edges are well localized. Therefore, only the limit condition has to be tested ($= v_d$): $V_{\text{non-deloc.}} \leq V_{\text{existence}} = v_d$.

Finally, whatever $v_d > 0$, regularization filter h will not delocalize the transition if $V_{\text{non-deloc.}} \leq V_{\text{existence}}$

$$\frac{h'_{-d}}{h'_0} \leq \frac{h_{-d}}{h_0} \quad \forall d \geq 1 \tag{8}$$

with $h' = f$ $(h'_k = h_k - h_{k-1})$. Fig. 2(c) represents an example of (discrete) definition for h.

2) Additive Influence $(a_v > 0)$*:* Additive influence is due to a neighbor edge with amplitude of the same sign: $u_k = A_c h_k + a_v h_{k-d}$. As this influence is additive, it results in an increase of the response amplitude, and it tends to delocalize this response. Derivative at $k = 0$ gives the delocalization sign: $u'_0 = a_v h'_{-d}$. The amplitude at $k = 0$ is always positive, and the maximum response is moved toward $k \geq 0$. Therefore, generally speaking, distance between consecutive detected edges, stairs figuring, tends to be shrunk, due to this additive influence. No edge detection should occur as soon as a "non-delocalization condition" is invalidated [no maximum for $k \leq (d/2)$]. If only one extremum (a maximum in the specific case) is assumed to exist for each transition, A_c will be detected at $k = 0$ (with a close-by transition at distance d) if $u_0 > u_1$ or if $A_c(h_0 - h_1) > a_v(h_{1-d} - h_{-d})$ or even

$$-A_c h'_1 > a_v h'_{1-d}. \tag{9}$$

Knowing that $h'_k = -h'_{-k+1}$ and noting $v_d = (A_c/a_v)$, (9) can finally be written as $v_d > (h'_{1-d}/h'_0)$. This condition is mandatory but not sufficient because delocalization or non-detection can occur for a closer transition. This condition has to be progressively completed as d is decreasing ($d = 2$ being the minimum value, closer edges cannot be resolved): $v_d > (h'_{-1}/h'_0) > (h'_{-2}/h'_0) > (h'_{-3}/h'_0) \ldots$ A regularizing filter h fulfilling this condition is monotonically decreasing along axis $-k$. If, in another situation, v_d is too small, the edge A_c, being not detected, will not be delocalized. As the response will be monotonically decreasing from d up to 0, the transition of amplitude a_v occurring at d will be the only one to be detected. Therefore, the mandatory and sufficient condition for detecting without delocalizing an edge under additive influence is as follows:

$$\frac{h'_{-1}}{h'_0} > \frac{h'_{-2}}{h'_0} > \frac{h'_{-3}}{h'_0} \ldots \tag{10}$$

If this condition is verified, the edges are either detected and well localized or not detected.

IV. Optimal Solutions

Numerous studies have been already conducted to provide optimal solutions with respect to precise criteria (localization, SNR, multiple responses) dealing with edge detection in continuous or discrete image processing. We propose, in the following part, the optimal filters fulfilling the above-described criteria in discrete domain: edge localization under noise influence (results already known), edge localization under additive and subtractive influences (new results). A conclusion summarizing in a table the properties of various existing regularizing filters is finally provided.

A. Localization Against Noise

Filters f minimizing denominator in the Canny's discrete criterion [9], [14] have by-parts constant and identical (without considering sign) derivatives. Filter impulse response f can be decomposed into straight segments having the same slope (without considering sign). The criterion is then optimally verified by a "triangle function" as in Fig. 3. This function is not equal to zero at $k = 0$ for the step response to have only one maximum.

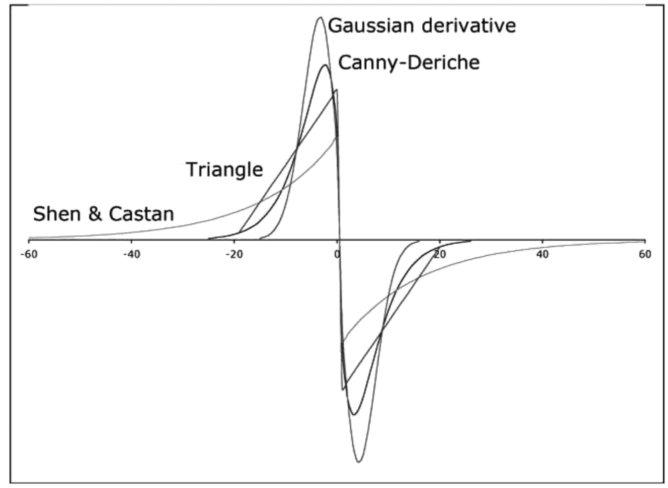

Fig. 3. Impulse responses of derivative filters used to compare localization properties. Filter scale parameters have been chosen to keep the same non-detection limit $(d = 10)$ under additive influence.

TABLE I
Properties of Classical Filters

f	satisfied Canny criterion*	delocalization† influence -	delocalization† influence +
Gaus. derivative	-	≤ 0	≥ 0
Canny-Deriche	SNR $\times$ Loc.	≤ 0	≥ 0
Shen & Castan	-	0	0
Demigny	SNR $\times$ Loc.	≤ 0	0
box	SNR	≤ 0	≥ 0
triangle	Localization	≤ 0	0

* The satisfied criteria defined by Canny are indicated: Localization, SNR and the product criterion (SNR $\times$ Loc.)
† The sign of the delocalization is given in function of the influence kind in the two last columns. Subtractive influence (-) corresponds to a two steps configuration with opposite slopes. Additive influence (+) corresponds to a two steps staircase configuration.

B. Localization Against Subtractive Influence

As it is easy to verify, exponential function is a solution of (8): $\begin{cases} h_{k\geq 0} = e^{-sk} \\ f_{k\geq 0} = -se^{-sk} \end{cases}$. This solution is optimal for it verifies the equality boundary case. Those discrete filters are formally identical to the continuous one defined by Shen and Castan [15]. Cord *et al.* [16] observed that the Shen and Castan filter never delocalizes two close edges in subtractive influence. Filters of higher decreasing rate are of no theoretical interest.

C. Localization Against Additive Influence

In order to fulfill (10), the derivative filter (derivative of regularizing filter) has to be strictly monotonically decreasing between $k = 0$ and $k = -L + 1$. Neither the Canny-Deriche [17] detector nor the classical Gaussian derivative does verify this condition. However, the "triangle filter" and, more obviously, the exponential one fulfill the non-delocalization criterion under additive influence.

D. Examples

The properties of some edge detector filters from various authors [9], [14], [15], [17] are summarized in Table I. Fig. 4 illustrates the delocalization phenomena, d varying, under subtrac-

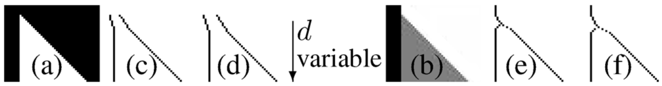

Fig. 4. Images illustrating a wide range of values for d $(a_v = \pm A_c)$ in (a) subtractive influence and (b) additive influence. Detection by Canny–Deriche and Gaussian derivatives in (c, d) subtractive influence and (e, f) additive influence.

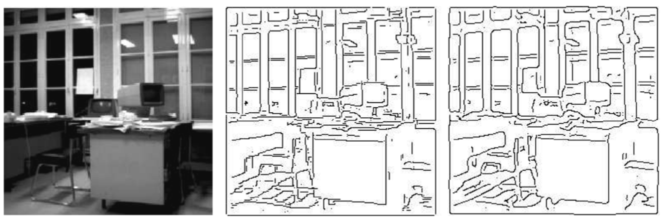

Fig. 5. (Left) Original image. (Center) Detection by Shen and Castan filter $(s = 0.4)$. (Right) Detection by Gaussian (derivative) filter $(\sigma = 2.2)$. Same SNR, Threshold $= 2$. The last image presents clearly edge delocalization.

tive and additive influence for edge detectors of Canny–Deriche and Gaussian derivative families. Finally, if subtractive influence is not an issue, then the "triangle filter" is the optimal solution according to noise-localization constraint.

E. Application to Real Images

Fig. 5 shows contour delocalization of the desk image processed with the Shen and Castan filter as well as with the Gaussian derivative filter. One used the classical 1-D separation approach where derivation occurs along the columns and smoothing (regularization) along the rows and vice-versa; then maxima localization followed by a threshold procedure is performed, resulting on the edge image. Images clearly show that the Gaussian filter is not robust against mutual influence and that contours are slightly moved away (the filters have the same SNR). The results obtained are rigorous and exact for vertical or horizontal sharp edges. However, for tilted edges, they are only approximate, although they lead to results of very good quality and better than those obtained if the introduced criteria are not respected. In the case of inclined contours, the regularization filter (applied according to the direction perpendicular to that of the detection filter) transforms sharp contours into ramp edges whose extent is function of the filter and of the orientation. This phenomenon can be taken into account according to the introduced method. It is, however, clear that the results will be more difficult to interpret and to implement in an effective algorithm. The best solution undoubtedly consists in processing each contour according to its direction (locally given initially). The problem of ramp-like edges is analogous, and it is not taken into account in this letter. However, in accordance with previously presented results [18], the optimum detection filter will be parameterized by the ramp-width, and thus, only one filter will never be optimum for all the ramp-widths and possible orientations of contours.

V. Conclusion

This letter highlights in a discrete approach the importance of the filter shape on weakly noisy images. Generally speaking, whatever the influence of neighbor edges (additive or subtractive), the Shen–Castan detector will be the only one to keep the detected edges at the right place. Clearly, this property can be of prime importance in many artificial vision applications, such as dimensional control, for instance.

It is also to be noted that in any real optical device, the PSF limits the bandwidth and can be considered as an intrinsic regularization filter. This PSF is very often modeled, at the first order, by a Gaussian (or a $sinc^2$ for a simple hole); as demonstrated in the previous parts, it can induce distortions on close by edge profiles resulting from delocalization of transition under additive or subtractive influence. In this letter, edge detectors able to localize correctly and detect edges even when they appear closely, as in angles, for instance, have been pointed out. Even if in the way of searching for universal ideal edge detectors, many other important criteria must be taken into account (sensibility to signal-over-noise ratio, multiple responses suppression, isotropic detection, etc.), the practical consequences for real image processing of the problem addressed in this letter deserve this special attention.

Finally, non-delocalization is preserved for high-order derivatives, which are useful for zero crossing detection.

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